BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D.C. 20024

SUBJECT: Rapid Determination of Radiation Dose in Low Altitude Orbits
Case 630

DATE: October 28, 1968

FROM: J.S. Ingley

ABSTRACT

Spacecraft orbiting the earth with apogees below 700 nautical miles encounter significant charged particle fluxes primarily in passage through the South Atlantic anomaly. Using this fact, this paper describes a method by which the radiation dose received by astronauts in low earth orbit can be rapidly calculated by hand using the altitude of the orbit at the latitudinal center of the anomaly as the critical parameter. Results are within a factor of 2 of machine calculations and in most cases are accurate to better than 50%.

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MEMORANDUM FOR FILE

The calculation of radiation dose received by astronauts in earth orbit requires the determination of electron and proton fluxes at suitable intervals during the flight and the integration of these fluxes over time. Because of the complicated nature of the trapped radiation belts it is necessary to carry out the calculations with the aid of a digital computer using suitable models of the trapped radiation environment such as those of Vette et al[1,2]. Given the initial orbital parameters, the Bellcomm Orbital Flux Program (BOFP) will calculate the electron and proton fluxes impinging on a spacecraft in earth orbit and integrate these fluxes over time. However, for a study involving a large number of different orbits, this process can be quite time consuming both in computer time and man-hours.

Radiation dose estimates for the ATM alternative mission study required the calculation of proton and electron doses for a large number of low altitude (apogee $\leq$ 700 NM) elliptical and circular orbits at various inclinations. In this altitude range most of the dose is received in passage of the spacecraft through the South Atlantic or Brazilian anomaly. This is a region of space where the earth's magnetic field dips relatively low and the trapped particles make their closest approach to the surface of the earth. For example, in a 24 hour period a spacecraft in 300 NM circular orbit at 30° inclination (orbital period $\sim$ 1.5 hours) will pass through the anomaly region 5 or 6 times in a 9 hour period spending 15-20 minutes in the high flux region on each pass. The remaining 15 hours will be spent in flux free regions of space as the rotation of the earth carries the anomaly region away from the plane of the orbit.

Using the fact that the dose is produced by passage through the anomaly region, it is possible to estimate the dose for low altitude orbits within a factor of 2 by rapid hand calculation using the altitude of the orbit at the latitudinal center of the anomaly region as the critical parameter. The results that follow will be restricted to orbits with apogees $\leq$ 700 NM. Above this altitude the spacecraft may encounter significant particle fluxes outside of the anomaly region, so that the altitude at the anomaly is no longer a good parameter for determining the dose.

In the following discussion the definition of the symbols is as follows:

r = geocentric radius of orbit.

Θ = polar angle in plane of the orbit ($\Theta = 0$ at perigee).

λ_p = geocentric latitude of perigee point.

λ_a = geocentric latitude of the center of the South Atlantic anomaly (= 23°).

Θ_p = in-plane angle from perigee to λ_a.

r_p = r at perigee.

i = inclination of orbit to earth's equator.

R_E = average radius of earth.

ε = eccentricity of orbit.

α = angle from the meridian plane to the orbital plane at perigee measured eastward from north to south.

With these definitions the orbit of the spacecraft is given by

$$r = \frac{r_p \, (1+\varepsilon)}{1 + \varepsilon \cos \Theta} \tag{1}$$

The altitude of the spacecraft is given by

$$h = r - R_E, \tag{2}$$

where we have ignored variations in the earth's radius with latitude. Although this introduces errors of $\pm$ 5NM in h, it is unimportant for the factor of 2 estimates of radiation dose we are trying to achieve here. The altitude of the spacecraft as it crosses λ_a is given by

$$h_A = \frac{r_p \, (1+\varepsilon)}{1 + \varepsilon \cos \Theta_p} - R_E. \tag{3}$$

θ_p can be determined from the equation (see Ref. 3)

$$\cos(90°+\lambda_a) = \cos(90° \pm \lambda_p) \cos \theta_p + \sin(90° \pm \lambda_p) \sin \theta_p \cos\alpha \qquad (4a)$$

where α is given by

$$\cos(i) = \cos (90°-\alpha) \sin (90° \pm \lambda_p). \qquad (4b)$$

The + or - sign is used depending on whether the perigee point is in the southern or northern hemisphere respectively. A little algebra gives the final result,

$$\cos\theta_p = \frac{+ \sin\lambda_p \sin\lambda_a \pm \sqrt{(\sin^2 i - \sin^2\lambda_a)(\sin^2 i - \sin^2\lambda_p)}}{\sin^2 i} \quad \text{(perigee in the south)} \qquad (5a)$$

$$\cos\theta_p = \frac{- \sin\lambda_p \sin\lambda_a \pm \sqrt{(\sin^2 i - \sin^2\lambda_a)(\sin^2 i - \sin^2\lambda_p)}}{\sin^2 i} \quad \text{(perigee in the north)} \qquad (5b)$$

Several distinct classes of solutions exist and are listed in Table I below. For class III the altitudes will of course be equal if the orbit is circular (ε=0).

TABLE I

CLASSES OF SOLUTIONS FOR COS θ_p

CLASS	SOLUTIONS
I. $\lambda_a > i$	No real solutions exist since orbit cannot cross latitude of the anomaly.
II. $\lambda_a < i$, $\lambda_p = i$	Orbit intersects latitude of anomaly twice at equal altitudes.
III. $\lambda_a < i$, $\lambda_p \neq i$	Orbit intersects latitude of anomaly twice at different altitudes.
IV. $\lambda_a = i$	Orbit tangent to latitudinal plane or anomaly.

Figure 1 is a plot of total magnetic field contours at 350 and 450 KM in the South Atlantic anomaly taken from reference 4. As can be seen, the anomaly is centered at about 23° south latitude and 47° west longitude. The rotation of the earth will cause the orbital plane to intersect the anomaly region twice per day if $i > \lambda_a$. During each period when the orbital plane intersects the anomaly region, the spacecraft will pass through the anomaly several times as described previously. Figure 2 is plot of omnidirectional proton flux (E > 34 MeV) impinging on a spacecraft in orbit for one sweep of the orbital plane through the anomaly region (during which time the spacecraft orbits the earth 5 or 6 times) versus altitude of the orbit at λ=23°. Figure 3 is the same thing for electrons (E > 0.5 MeV). The closed circles are results from calculations done on BOFP (Bellcomm Orbital Flux Program) on a variety of circular and elliptical orbits. The electron results are projected to December 1968 taking into account decay of the Starfish electrons. The equations for the solid lines are:

$$\Phi_e\ (E > 0.5\ \text{MeV}) = 4.3 \times 10^7 e^{\frac{h_A}{158}}\ e/cm^2 \qquad (6a)$$

and

$$\Phi_p\ (E > 34\ \text{MeV}) = \begin{cases} 1.6 \times 10^5 e^{\frac{h_A}{126}}\ p/cm^2 & (h_A > 300\text{NM}) \\ 5.4 \times 10^3 e^{\frac{h_A}{52}}\ p/cm^2 & (h_A \leq 300\text{NM}). \end{cases} \qquad (6b)$$

The dashed lines are a factor of 1.5 above the solid lines. Below 300 NM the electron flux is very sensitive to the inclination of the orbit, and figure 3 is only accurate for $i \geq 50°$. For orbits with $i < 50°$ figure 3 can serve as an upper limit, but the acutal flux may be up to a factor of ten lower.

In order to estimate the skin dose to an astronaut the energy spectrum of protons and electrons and the shielding configuration of the spacecraft must also be known. A solid angle shielding breakdown of the lunar module and command module were used with the proton and electron spectra from the BOFP calculations to calculate skin dose for the actual orbits shown as closed circles in figures 2 and 3. The results are given by equations 7a, 7b, and 7c. Equation 7a gives electron skin dose for lunar module and spacesuit (LM+SS) configuration as a function of electron flux above 0.5 MeV, and equations 7b, 7c proton skin dose in the LM+SS and the command module (CM) as a function of proton flux above 34 MeV.

LM+SS Electron Dose:

$$D_e \text{ (LM+SS)} = 0.52\ \Phi_e \text{ rads} \tag{7a}$$

LM + SS Proton Dose:

$$D_p \text{ (LM+SS)} = \begin{cases} (0.26\ \Phi_p - 1.0) \text{ rads} & \Phi_p \geq 7 \\ 0.12\ \Phi_p \text{ rads} & \Phi_p < 7 \end{cases} \tag{7b}$$

CM Proton Dose:

$$D_p\text{(CM)} = 0.024\ \Phi_p \text{ rads} \tag{7c}$$

The units of Φ_e are 10^8 e/cm^2 above 0.5 MeV, and the units of Φ_p are 10^6 p/cm^2 above 34 MeV. Electron skin dose in the command module is negligible. The non-linear behavior of equation 7b results from spectral changes in the proton flux with altitude in the anomaly. Equations 7a, 7b, and 7c can be used in conjunction with the fluxes from figures 2 and 3 to yield skin dose versus altitude of passage through the anomaly for the CM and LM+SS. Electron skin dose for the LM without spacesuit is about 3-4 times higher. Proton skin dose is essentially the same.

SUMMARY

Proton and electron skin doses for low altitude (<700 NM) elliptical or circular orbits can be quickly estimated by the following procedure:

1. Given r_p, ε, i, λ_p, and R_E.
2. Calculate $\cos \Theta_p$ from equations 5a or 5b with $\lambda_a = 23°$.
3. Calculate values of h_A from equation 3.
4. Determine Φ_p (E >34 MeV) and Φ_e (E> 0.5 MeV) from figures 2 and 3 or equations 6a and 6b for each value of h_A (see Table I). Add these to get flux per day. For

   $\lambda_a = i = 23°$, treat as two intersections at equal altitudes (i.e., flux/day equals twice graph value).
5. Determine daily skin dose in CM and LM+SS from equations 7a, 7b, and 7c.

6. If i is less than 23° an upper limit to the dose can be obtained by assuming i = 23°.

The estimates should be within a factor of 2 of computer calculations made with BOFP and will probably be within 50%. Comparison of computer calculations with the graphical estimates (in parentheses) for several typical orbits is illustrated in the table below. The success of this technique reemphasizes the well known fact that radiation doses in low earth orbit are acquired in brief bursts (15-20 minutes) during passage through the South Atlantic anomaly and at much lower rates over other portions of the orbit, provided the altitudes are lower than perhaps 700 NM.

TABLE II

COMPARISON OF GRAPHICAL ESTIMATES OF DOSE WITH COMPUTER CALCULATIONS

PERIGEE	APOGEE	INCLINATION	λ_p	PROTON DOSE (RAD)		ELECTRON DOSE (RAD)
				CM	LM+SS	LM+SS
150 NM	400 NM	63.5°	0°	.091(.080)	.39(.37)	1.9(2.7)
150 NM	400 NM	50°	0°	.12(.077)	.49(.35)	1.6(2.5)
150 NM	675 NM	50°	30°N	.58(.76)	5.4(7.1)	15.(15.5)

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Attachment
Figures 1,2,3

REFERENCES

1. Vette, J.I., "Models of the Trapped Radiation Environment," Volume I, NASA SP-3024, 1966.

2. Vette, Lucero, and Wright, "Models of the Trapped Radiation Environment", Volume II, NASA SP-3024, 1966.

3. James, D.B., Leibowitz, M.A., Schulte, H.J., "The Radiation Environment of Apollo", Interim Report, Bellcomm 1963.

4. Cain, J.C., Langel, R.A., Hendricks, S.J., "Magnetic Chart of the Brazilian Anomaly - A Verification", Geomagnetism and Aeronomy, August 1, 1968, pp. 84-87.

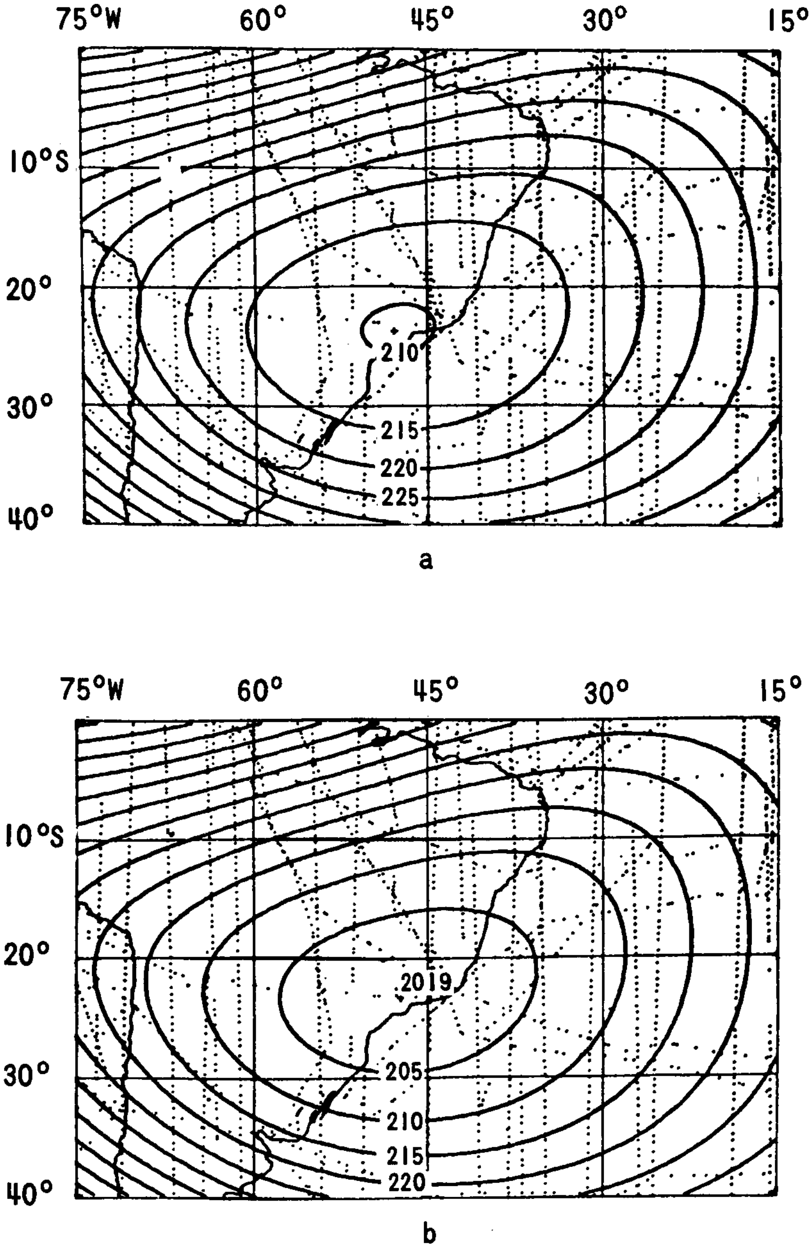

FIGURE I - CONTOURS OF TOTAL FIELD F OVER THE BRAZILIAN ANOMALY COMPUTED FROM GSFC(12/66) COEFFICIENTS. POSITIONS OF DATA AFTER 1960.0 USED IN DETERMINATION OF COEFFICIENTS ARE PLOTTED AS DOTS.
a) 350 KM ALTITUDE; b) 450 KM ALTITUDE

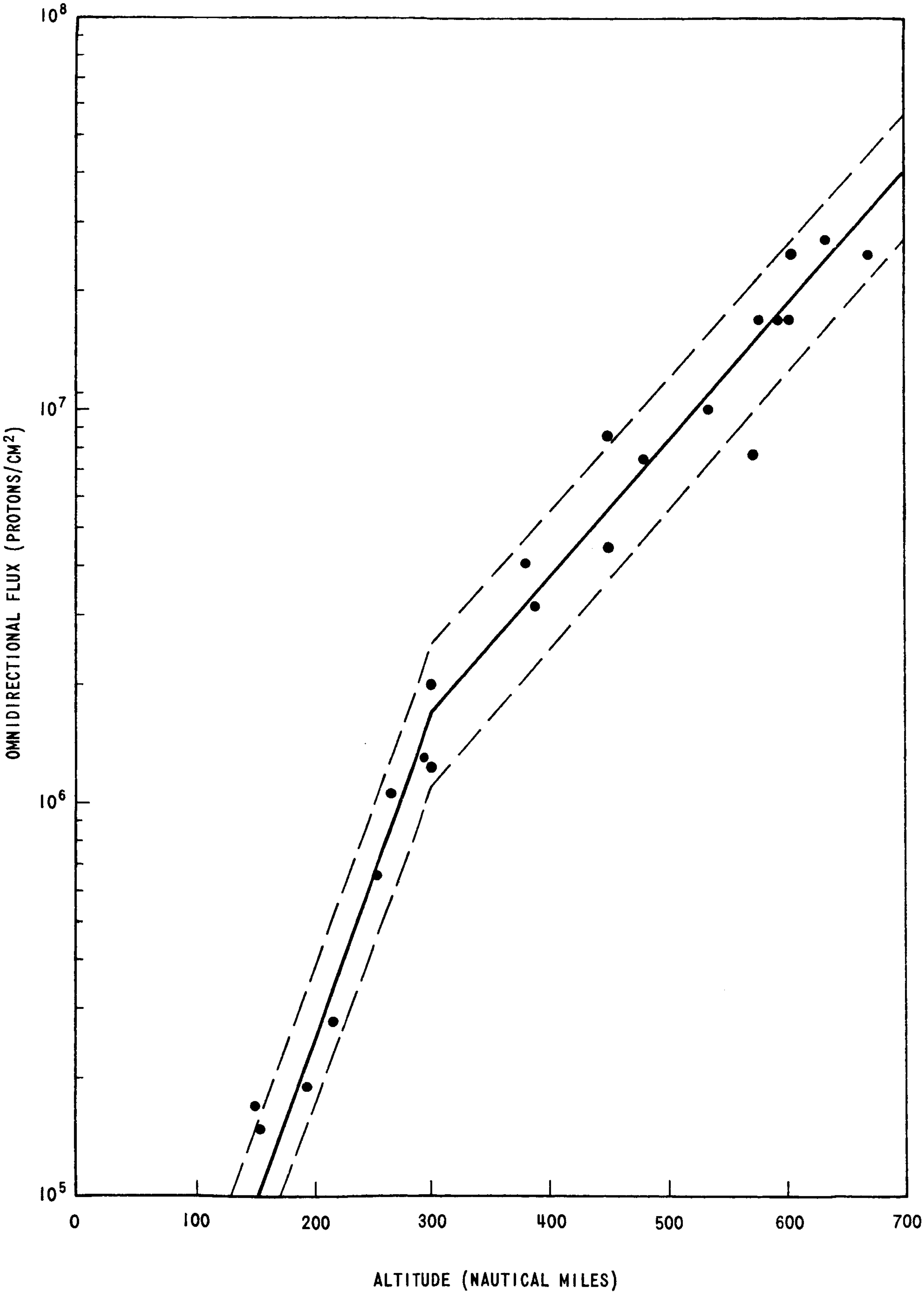

FIGURE 2 - OMNIDIRECTIONAL PROTON FLUX FOR ONE SWEEP OF ORBITAL PLANE THROUGH ANOMALY REGION VERSUS ALTITUDE OF ORBIT AT $\lambda = 23°S$

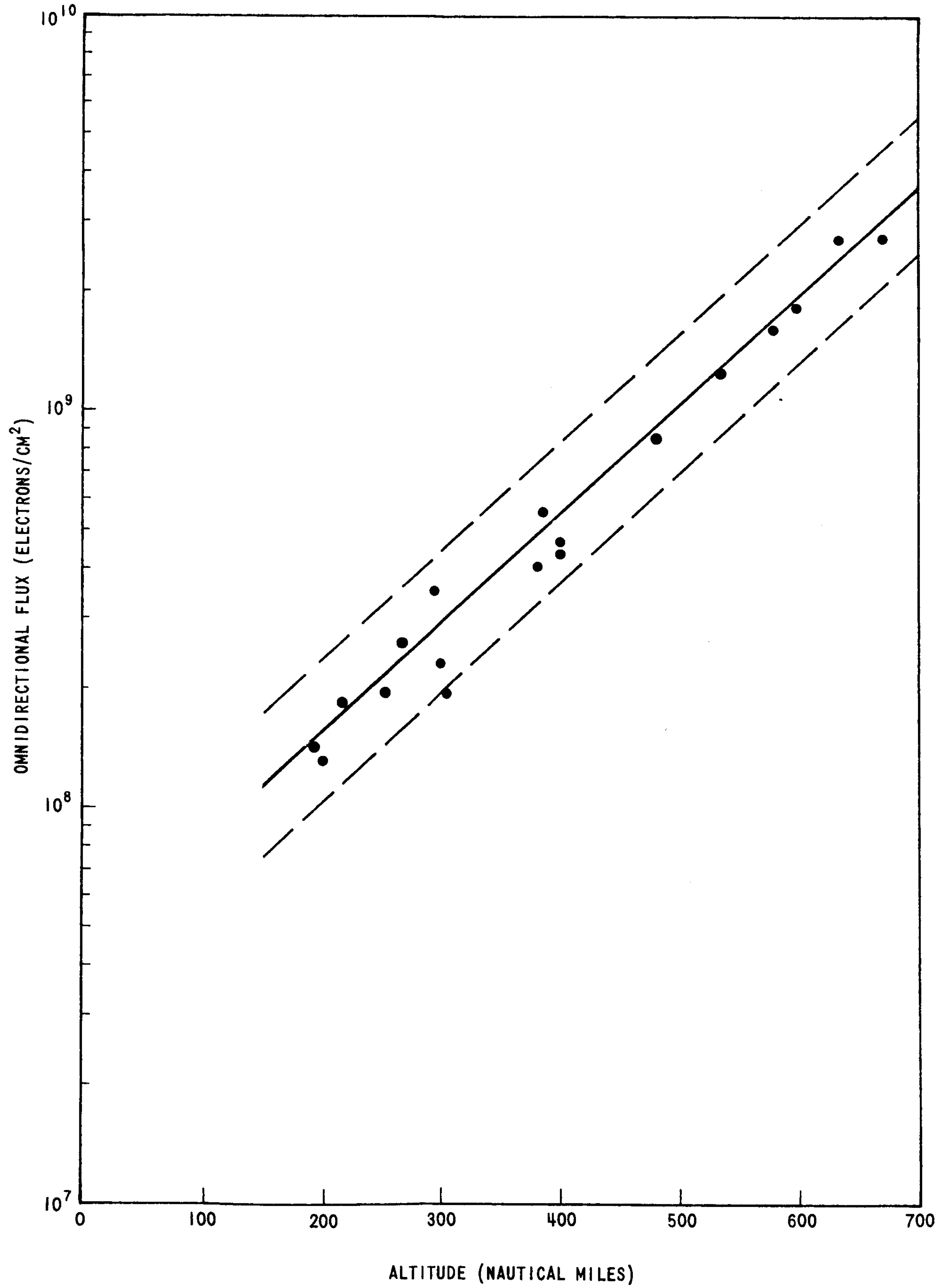

FIGURE 3 - OMNIDIRECTIONAL ELECTRON FLUX FOR ONE SWEEP OF ORBITAL PLANE THROUGH ANOMALY REGION VERSUS ALTITUDE OF ORBIT AT $\lambda = 23°S$